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A MONTE CARLO EVALUATION OF TWO MULTIDIMENSIONAL SCALING METHODS FOR FITTING THE WEIGHTED EUCLIDEAN DISTANCE MODEL: INDSCAL vs. ALSCAL

by

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Abstract

Multidimensional scaling (MDS) methods INDSCAL and ALSCAL are compared using the Monte Carlo method. The criteria used were recovery of true distances, recovery of stimulus configuration and recovery of true weight structure. INDSCAL performs better than ALSCAL - except for a few cases.

Keywords: MDS, INDSCAL, ALSCAL, metric determinacy

I. Introduction

Multidimensional Scaling (MDS) may be defined broadly as a family of geometric models for multidimensional representation of data and a corresponding set of methods for fitting such models to actual data (Carroll and Arabie, 1980). While several different geometric models comprise the family of models referred to in this definition (see Carroll and Arabie, 1980) for a thorough review of these models), a much narrower definition would limit the term to spatial distance models for similarities, dissimilarities or other proximities data. Spatial distance models in MDS provide us with a set of tools for recovering the underlying structure which is "hidden" in the data, and for generating a spatial representation of the relationships among the set of n stimuli or objects designated as O_1, O_2, ..., O_n.

In practical terms, MDS allows us to make a picture or map of the information in the data. When the set of data is large, the practical value of MDS is clear, since in such cases a map is generally easier and more informative to look at than the data itself. In spatial distance models, data are organized so that objects which are similar to one another (in some empirical sense) are placed close together in the space, and objects which are dissimilar to one another are placed far from each other in the space (Spence, 1972). Thus, the primary objective in MDS for spatial distance models is to map the objects in a multidimensional space in such a way that their relative positions in the space reflect the degree of proximity (similarity) between the objects.

Perhaps the easiest way to understand what MDS does is to consider the following problem familiar to MDS users (Kruskal and Wish, 1969). Let us assume that one is shown the map of the United States and is asked to construct the table of intercity distances. This is an easy task. With the use of straightedge and a compass, one could easily produce the distance between the cities. But if the problem is reversed, say if one is given the set of distances, such as those found in the bottom of maps, and asked to recreate the map itself, then this proves to be a far more difficult exercise, though it can be solved with the use of a straightedge and a compass in two dimensions. MDS is a method for solving this reverse problem without a straightedge and a compass (Kruskal and Wish, 1978). In actual application settings the problem is exceedingly more complex because the data usually contain error or "noise" and it is rarely known in advance whether a two-dimensional solution will be adequate (Dillion and Goldstein, 1984).

II. Need for the Study

During the last two decades, studies on MDS were especially concerned with MDS procedures which operated on two-way data matrices (Stenson and Knoll, 1969; Klahr, 1969; Young, 1970; Wagenaar and Padmos, 1971; Sherman, 1972; Isaac and Poor, 1974; Spence and Domoney, 1974; Spence, 1972; Girard and Cliff, 1976) to the exclusion of three-way scaling methods. With the variety of three-way methods for fitting the weighted Euclidean (INDSCAL) model, and with little known with respect to the goodness of their solutions for particular types of data sets, a user is apt to choose a method based on its accessibility. The purpose of this study was to investigate the behavior of the two most widely used three-way MDS algorithms that have been developed for fitting the weighted Euclidean model: INDSCAL designed specifically for metric data and ALSCAL designed specifically for nonmetric data. While highly popular, relatively little is known about the properties of these techniques and the validity of the solutions they provide under different conditions.

Accordingly, the purpose of this study was to investigate by Monte Carlo methods, the validity of INDSCAL and ALSCAL to recover the true structure inherent in different sets of simulated data as a function of number of subjects, number of stimuli, error level and type of monotonicity (transformation).

Recovery of the true structure of the data, the criterion to be used in this study to compare the properties of the INDSCAL and the ALSCAL algorithms, is important as it reflects the validity of a solution.

III. Design and Data Generation

In order to compare the ability of three-way metric INDSCAL and three-way nonmetric and metric versions of ALSCAL to recover the true structure of proximity data, a simulation study was employed. The general design of the simulation study was as follows: (a) generation of simulated proximity measures which have a known structure for a sample of individuals, (b) analysis of these proximity measues via the INDSCAL and the ALSCAL procedures, (c) a study of the degree to which each of these procedures is able to recover the known structure of the data.

In generating the various sets of simulated data, four characteristics of the data were controlled as independent variables: (a) number of subjects; (b) number of stimuli, (c) amount of error, and (d) type of monotinicity (transformation). Because of the differences between the INDSCAL fitting criteria, it would be very difficult, if not impossible, to determine equivalent convergence criteria for each algorithm; i.e. values that required rather small changes in order for the literature process to terminate (MacCallum, et.al., 1978). Because ALSCAL uses SSTRESS while INDSCAL uses VAF as the criterion for convergence only three of the four possible measures of recovery of the structure of the data were employed in this study. These were (a) RSQ, the squared correlation between true and recovered distances among stimuli, or proportion of variance in the true distances accounted for by the reconstructed distances, (b) δ, the root mean squared deviation between true and recovered projections of the n-th stimuli on the r-dimensions, and (c) φ, 1.0 minus the mean of the cosines of the angles between the true and recovered subject weight vectors. Better levels of recovery are represented by higher values of RSQ and lower values of δ and φ.

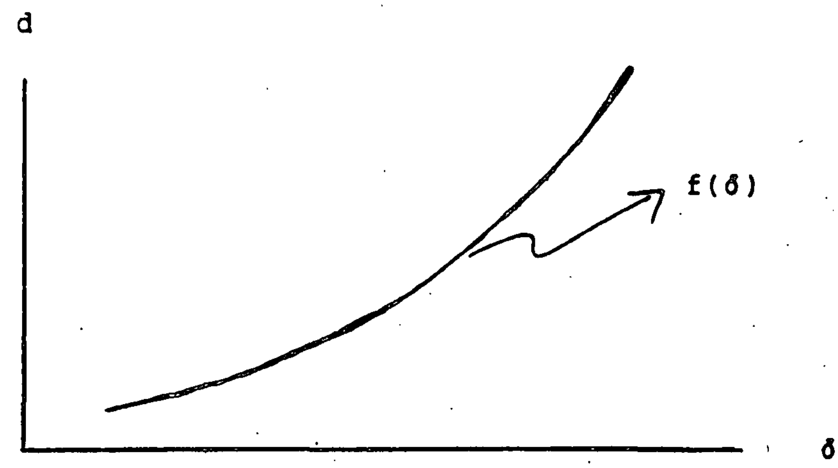


Figure 1. d^p where $p = 2$

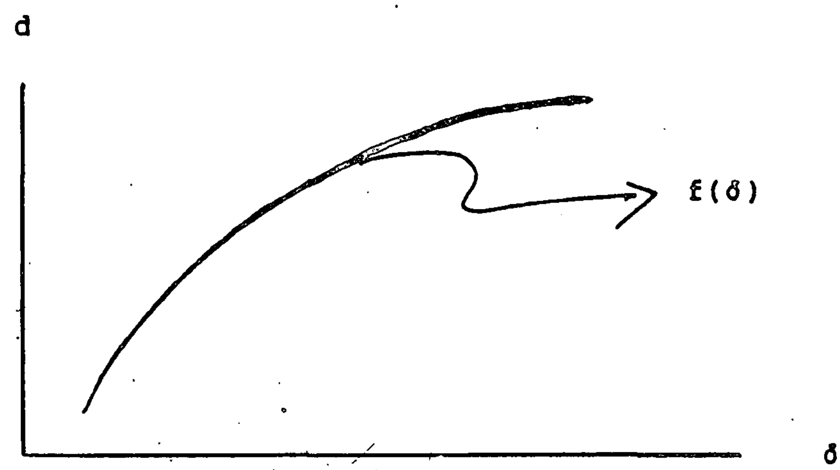


Figure 2. log d

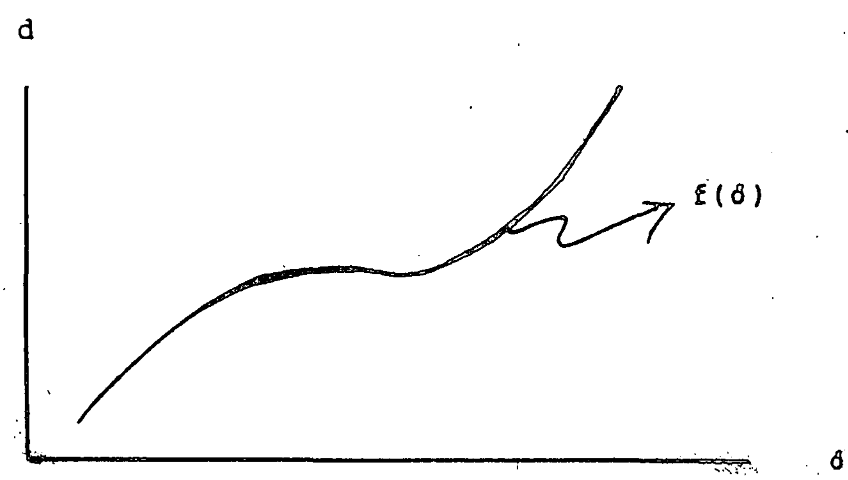


Figure 3. d^p where $p = 3$

The design incorporated 15 and 30 subjects and 12 and 20 stimuli. These values were chosen as being typical of those employed in empirical reserarch utilizing MDS. The distribution of errors were $N(0,\delta_e)$ with δ_e equal to 0.10 and 0.70, corresponding to small and large amounts of error, respectively (Girard and Cliff, 1976). Finally, five monotonic transformation of the data were specified. To satisfy the metric assumptions of INDSCAL, true distances, d, were transformed according to d^p, where p = 1(linear). To satisfy the assumptionss of ordinal ALSCAL, true distances within each slice were rank ordered, rank(d). The three remaining transformations representing a variety of possible nonlinear monotonic transformations were d^2 (a concave transformation), log d (a convex transformation), and d^3 (a concave to convex transformation). Figures (1), (2) and (3) present the three nonlinear monotonic transformations employed in this study.

Note that Figure (1) is the concave transformation obtained by squaring the true distances; Figure (2) is the convex transformation obtained by taking the logarithm of the true distance; and Figure (3) is the cubic transformation formed by fitting the true distances to the polynomial

$$f(x) = x^3 - 2.55x^2 + 2.1675x \qquad (1)$$

Equation (1) was derived by evaluating the general polynomial equation at the median of the true distances.

For each fo the 40 (2x2x2x5) number of subjects x number of stimuli x error level x type of monotonic transformation, five replications of sets of stimulated proximities were generated, resulting in 200 such data sets. Each data set was analyzed by each of the metric INDSCAL and the metric and nonmetric ALSCAL procedures resulting in 600 analyses.

True configurations were generated to have two underlying dimensions. Twelve and twenty stimulus coordinates for each of the two dimensions were generated by selecting at random 12 and 20 points respectively from a uniform distribution over the unit square. Subject weight matrices, consisting of 15 and 30 subjects were generated for each of the two dimensions subject to the constraint that a moderate level of individual differences be maintained. Accordingly, individual weights varied betweedn 10 and 80 degrees from the positive horizontal axis in the two-dimensional space (macCallum, et. a., 1977). Moreover, to avoid large variance across individual with respect to distance from the origin in the weight space, the weights generated for each individual were rescaled so that their sum of squares is c_i, where c_i is a random number of lying in the interval 0.5 to 1.0. Accordingly, each c_i represents the squared distance of an individual from the origin in the weight space.

Based on these true subject weights and stimulus coordinates, true Euclidean distances were computed between all pairs of points within a given data set. For each individual, distances were normalized to unit variance. Distances were then transformed according to the five types of monotonicity (transformations), and small or large error was added.

IV. Evaluation of Recovery of True Structure

As mentioned earlier in this report, three aspects of recovery of structure of the data were studied.

1. Goodness of fit to True Distances. The first aspect of recovery which was investigated was the ability of both the INDSCAL and the ALSCAL methods to recover the true interpoint distances, the d_{ijk}. To measure this, distances were computed on the basis of both the INDSCAL and the ALSCAL solutions, and then these recovered distances were correlated across all stimulus pairs and individuals with the known true distances. The actual dependent variable employed was the square of this correlation, RSQ, which corresponds to the "index of metric determinacy" (Young, 1970) extended to a sample of individuals. Mathematically, RSQ is defined as

$$RSQ = \frac{m\,(\Sigma d_{ijk}\hat{d}_{ijk}) - (\Sigma d_{ijk})(\Sigma \hat{d}_{ijk})}{([m(\Sigma d^2_{ijk}) - (\Sigma d_{ijk})^2][m(\Sigma \hat{d}^2_{ijk}) - \Sigma(\hat{d}_{ijk})^2]^{1/2})} \qquad (2)$$

where d_{ijk} is the true distance between stimuli j and k for individual i; $\hat{d}_{ijk}$ is the recovered distance computed across the n(q-1)/2 stimulus pairs and m individuals.

2. Recovery of True Stimulus Configuration. The second aspect of recovery examined was the ability of both the INDSCAL and the ALSCAL procedures to recover the true stimulus configuration. To measure this property, the square root of the mean squared difference between the true and recovered projections, computed across stimuli and dimensions was

computed. This is defined as

$$\delta = \{ \sum_{j=1}^{n} \sum_{t=1}^{r} \frac{(x_{jt} - \hat{x}_{jt})^2}{nr} \}^{1/2} \qquad (3)$$

where, $\hat{x}_{jt}$ is the recovered projection of stimulus on dimension t. For perfect recovery of all stimulus coordinates, δ would be zero (macCallum, et al., 1977). The use of this recovery requires some important considerations. First, since the actual values of the true and recovered projections were compared, it was important that the scale properties of the true and recovered stimulus spaces be the same. For the true space, each stimulus dimension was constructed to have a mean of zero and a sum of squares equal to 1. Since the ALSCAL algorithm scales its solution so that the projections on each stimulus dimension have a mean of zero and a sum of squares equal to n, number of stimulus points, each of the points in the recovered space was divided by the square root of n to convert to a unit sum of squares. Since the normalization of dimensions in INDSCAL agrees with the normalization of the true space mentioned earlier, INDSCAL solutions did not need to be normalized. Secondly, it was important to take into account the possibility that the dimensions of the recovered stimulus space represented permutations and/or reflections of the dimensions of the true stimulus space. To allow for this possibility, the recovered dimensions were whenever necessary, rearranged and reflected so as to obtain in the optimal value of δ. This was done by computing δ for all possible permutations of the recovered ALSCAL and INDSCAL dimension which exhibited a negative correlation between true and recovered projections was reflected in the recovered space. By this procedure, the optimal arrangement of recovered dimensions, and corresponding measure of recovery of the true stimulus configuration was obtained for each set of simulated data (MacCallum, et. al., 1977).

3. Recovery of True Weights. The third aspect of recovery which was of interest was the ability of both the INDSCAL and the ALSCAL procedures to recover the true weights employed by the individuals. Following Takane, et.al., (1977), the development of an index of recovery of weights was approached from a geometric perpective. A given set of weights for m individuals on r dimensions is often represented geometrically as m points in r dimensions, with the coordinates of the points corresponding to the weights. To examine recovery of weights one could plot, in a single r dimensional space, two sets of points: one set corresponding to the true weights and one set corresponding to the recovered weights. Perfect

recovery would simply require that the m vectors defined by the recovered weights would each lie in the same direction as the m corresponding vectors defined by the true weights. Takane, Young, de Leeuw (1977) and MacCallum (1977) point out that for conditional data one cannot expect ALSCAL to recover true weights but only true weight ratios. Geometrically, this means that for conditional data ALSCAL cannot be expected to recover the true location of an individual point in the weight space, but only its angular orientation with respect to the coordinate axes. This phenomenon is also true of INDSCAL. Therefore, a measure of recovery of the weight structure is based on a ratio comparison of the angular orientation of the m true weight vectors and the m recovered weight vectors. This was accomplished as follows: first the r dimensions of the recovered weight space were rearranged so as to match the rearrangement of the recovered stimulus dimensions, (i.e., after necessary permutations and reflections); and for each of the individual, the cosine of the angle between the true weight vector and the recovered weight vector was computed, which was denoted as θ_{ij} then the mean of these cosines was computed, and the measure of recovery was defined as in MacCallum, et al., (1977) by

$$\varphi = 1.0 - \sum_{i=1}^{m} \frac{\cos\theta_i}{m} \qquad (4)$$

For perfect recovery of weight vectors, φ would be zero.

V. Data Analysis

Thus, three dependent variables were defined: RSQ or metric determinacy, measuring recovery of true distances; δ, measuring recovery of the true stimulus configuration; and φ, measuring recovery of the true weight structure. Values of these three indices were obtained for each of the 200 INDSCAL and 400 ALSCAL solutions.

To determine the effect of the independent variables on the three dependent variables (recovery measures) 3 separate 2x2x2x5x3 five-way univariate repeated measures analyses of variance were performed. Within this framework, the first four factors were between data sets, and the last factor was the within data sets, method of analysis (INDSCAL vs. ALSCAL-ORDINAL vs. ALSCAL-INTERVAL).

VI. Results, Discussion, and Recommendations

Before conducting the analyses of variance, redundancy among the three outcome measures was examined. The pooled within cell correlations among the dependent variables were as follows: for INDSCAL, $r_{(RSQ)\delta}$ = -.4576, $r_{(RSQ)\varphi}$ = -.3018, $r_{\delta\varphi}$=.4971; for ALSCAL-ORDINAL, $r_{(RSQ)\delta}$ = -.0890, $r_{(RSQ)\varphi}$ = .2686, $r_{\delta\varphi}$ = .6275; and for ALSCAL-INTERVAL, $r_{(RSQ)\delta}$ = -.0740, $r_{(RSQ)\varphi}$ = .2303, $r_{\delta\varphi}$ = .6054. Since most of the within cell correlations are low and only two are approximately .6, the three measures of recovery may be viewed as measuring fairly different aspects of recovery and therefore separate univariate analyses of variance will provide little redundancy of information. The results of the analyses of variance are summarized in Tables 1, 2, and 4 which contain the significance levels and omega squares (w^2) for all the three recovery measures. Omega square is a measure of effect size, i.e., proportion of variance explained. Because statistically significant results may not be of practical import, only those which are meaningful, i.e., those which explain about 5% or more of the variance (w^2 ~ .05) are discussed.

1. Index of Metric Determinacy (RSQ)

Table 1 shows the summary of the analysis of variance of RSQ, or index of metric determinacy. The table reveals that a three-way interaction (transformation by method by error level) was significant, $F(8,354)$ = 16.88907; p<.00001, which explained 7% of the within data sets variance. The significant two-way interactions are those between transformation and method (w^2=.11626), $F(8,354)$ = 27.64061, p<.00001, and between error level and method (w^2=.48258), $F(2,354)$ = 451.11101, p<00001. Tests of simple interaction effects yielded five significant two-way interactions, i.e., method by error level at each type of transformation: method by error at linear, $F(2,320)$ = 132.16, p<.00001; method by error at square, $F(2,320)$ = 50.39, p<.00001; method by error at cubic, $F(2,320)$ = 137.17, p<00001; method by error at rank, $F(2,320)$ = 94.77, p<.00001 and method by error at logarithmic, $F(2,320)$ = 126.95, p<.00001. Figures (4), (5), (6), (7), and (8) show the five simple interactions.

Figures (4) through (8) show that at each error level, the three methods differ at each type of transformation. Specifically, the Scheffe tests, p<.05 reveal that at small error, ALSCAL performs better than INDSCAL under the cubic, rank and logarithmic; while INDSCAL is superior to ALSCAL under the large error in all types of transformation. Table 1a shows the cell means and standard deviation of the RSQ measure.

Index of Fit for Stimulus Dimensions (δ)

A summary of the analysis of variance for this measure is presented in Table 2. Note that of the between data factors, only one main effect is significant, i.e., the number of stimuli which is explaining about 9% of the variance, $F(1,177) = 21.17216$, $p<.00001$. All the between data factors interactions were not significant. With respect to the within data factors, variance accounted for is substantial for method of analysis ($w^2 = .76529$), $F(2,354) = 628.25886$, $p<.00001$. Although the two-way interaction between number of stimuli and method is significant, variance accounted for is not meaningful, ($w^2<.01$), hence this is disregarded. The marginal means shown on Table 3 reveals the superiority of INDSCAL over ALSCAL in the δ measure. Also, the Scheffe tests, $p<.05$ show significant differences between INDSCAL and ALSCAL in both the metric and the nonmetric versions. Table 2a (see Appendix) presents the cell means and standard deviations of the measure.

Index of Fit for Subject Weights (φ).

Table 4 shows the results lof the analysis of variance of φ. Of the between data factors the two-way interaction between type of transformation and number of subjects accounts for about 5% of the variance, $F(4,177) = 3.51024$, $p = .00873$. Regarding the within data factors, method of analysis was significant accounting for about 57% of the variance, $F(2,354) = 350.48285$, $p<.00001$. There are 21 together six significant interactions, but the only one that is meaningful is the two-way interaction between error level and method of analysis which is explaining for about 5% of the variance, $F(2,354) = 30.49185$, $p<.00001$. Table 4a (see Appendix) shows the cell means and standard deviations of the φ measure. Figure (9) elucidates the meaning of the significant interaction.

Figure (9) shows that in the recovery of subject weights, INDSCAL is better than ALSCAL for both small and large error but its superiority to ALSCAL is enhanced under small error.

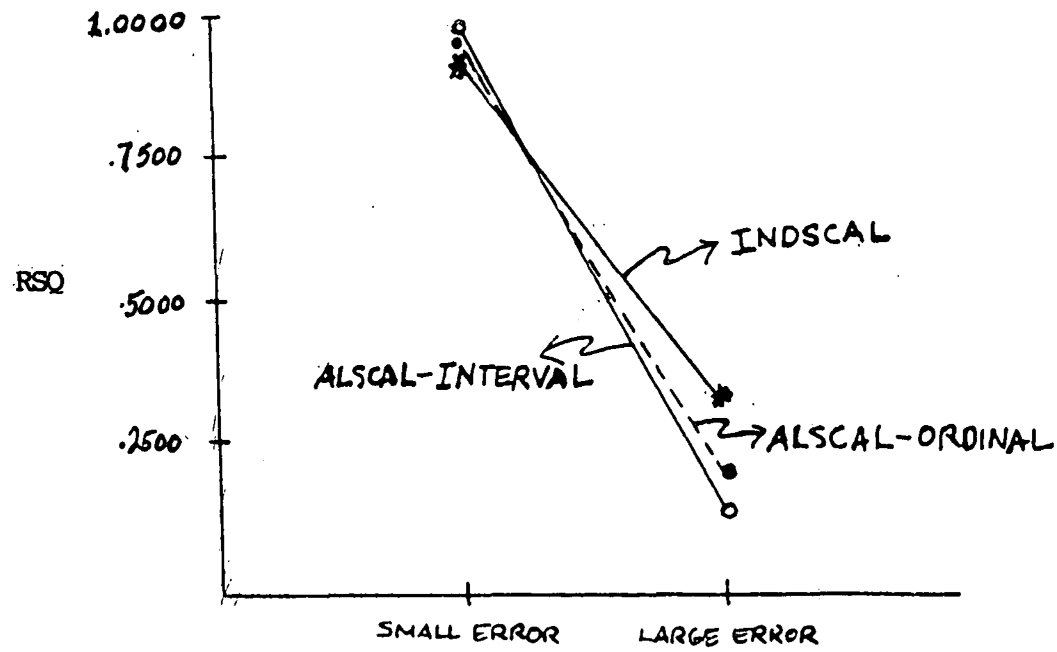


Figure 4. Effect of Method by Error Interaction at Linear on RSQ

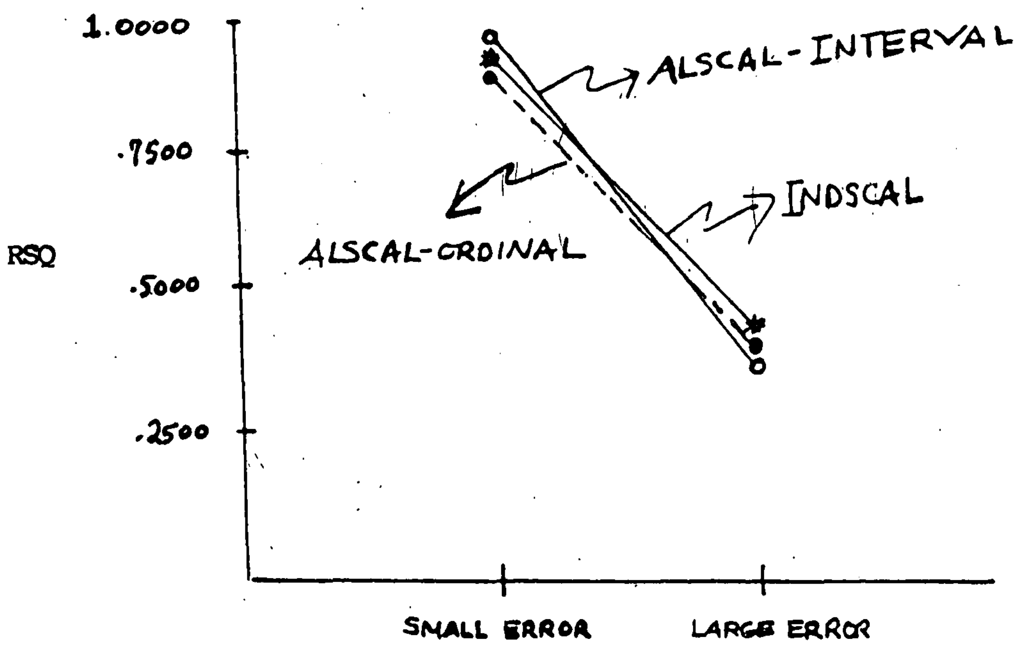


Figure 5. Effect of Method by Error Interaction at Square on RSQ

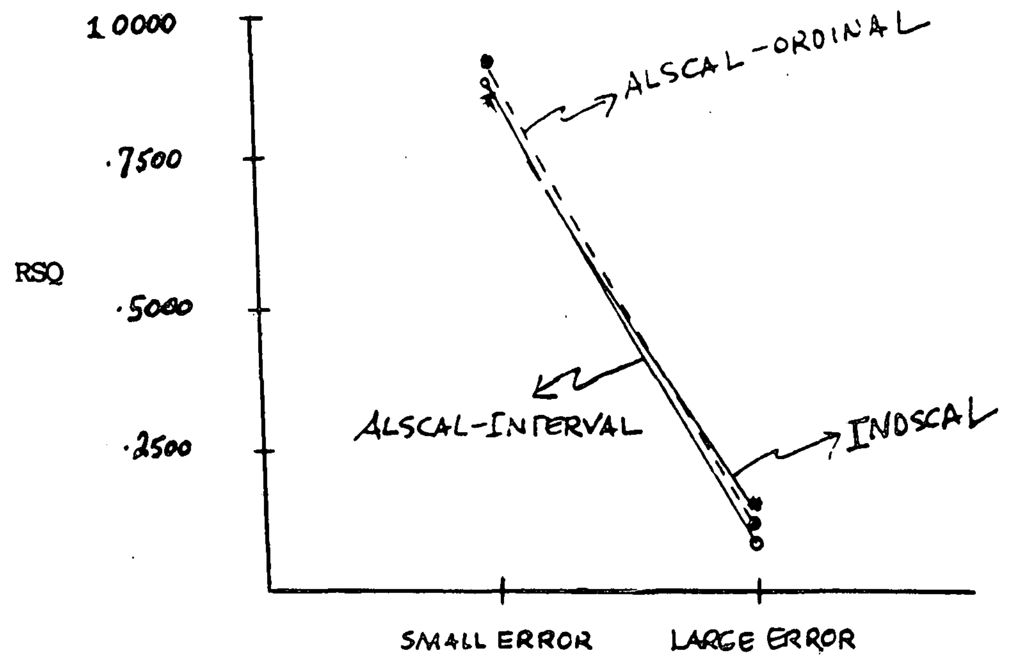


Figure 6 : Effect of Method by Error Interaction at Cubic on RSQ

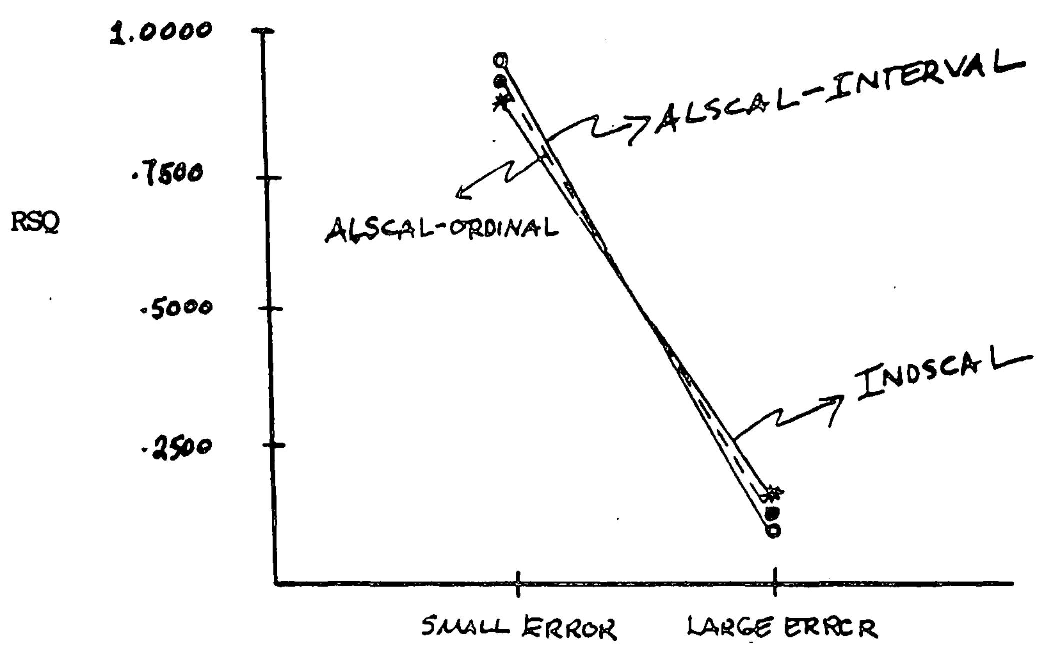


Figure 7 : Effect of Method by error Interaction at Rank of RSQ

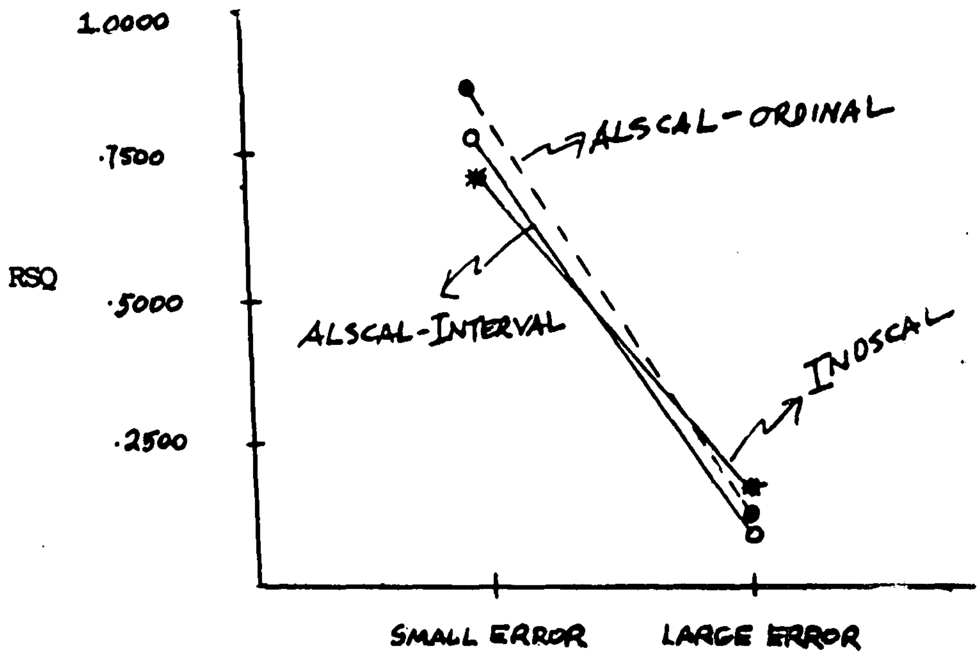


Figure 8 : Effect of Method by Error level Interaction at Logarithmic on RSQ

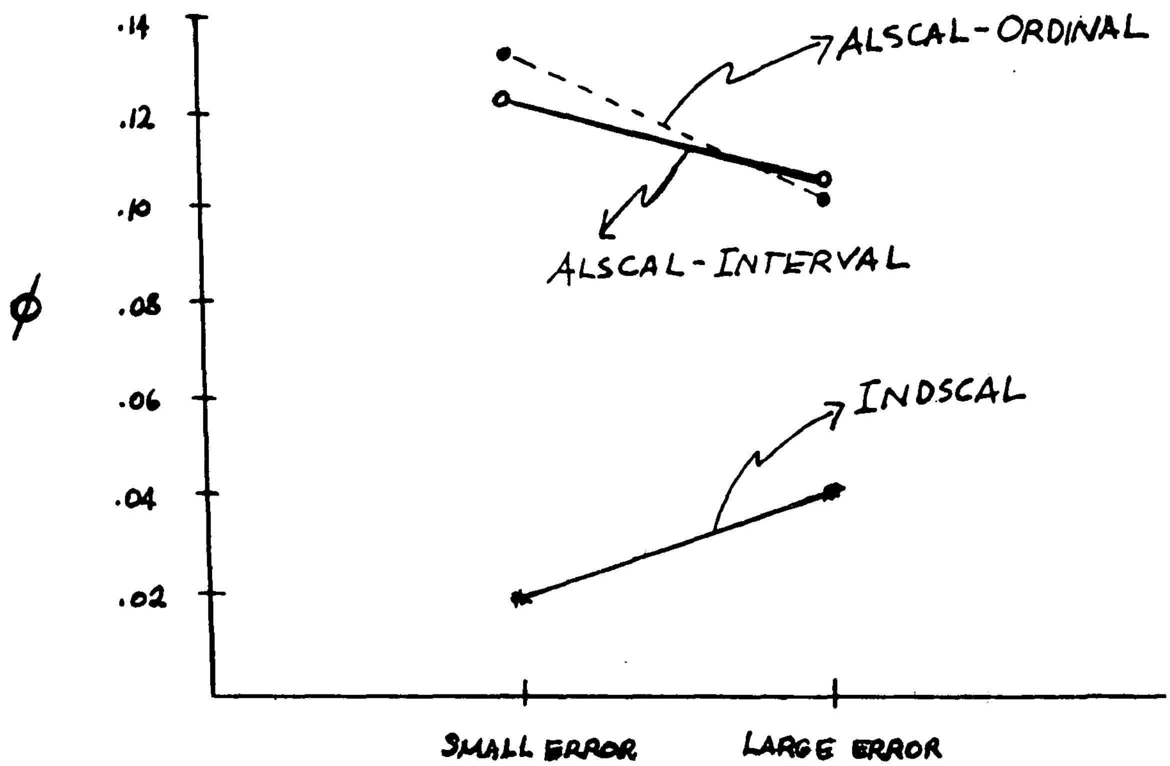


Figure 9 : Effect of Method of Analysis by Error Level Interaction on ϕ

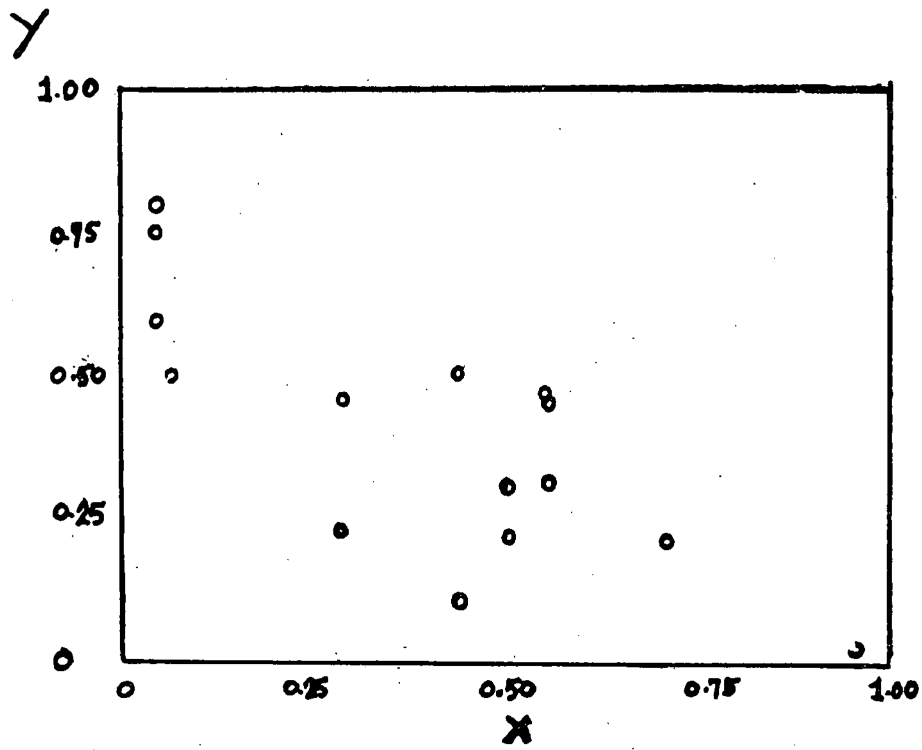
Discussion

Recall that better levels of recovery are represented by higher values of RSQ and lower values of δ and φ. Therefore RSQ values in the neighborhood of .80 and .90 found in the present study can be considered reasonably good fit. According to MacCallum, et.al., (1977), RSQ, δ or a φ in the neighborhood of .30 or .40 would not indicate accurate recovery of the true distances, true stimulus coordinates and true subject weights. Therefore, the values found in this study in all the three indices good recovery except for RSQ in the large error level.

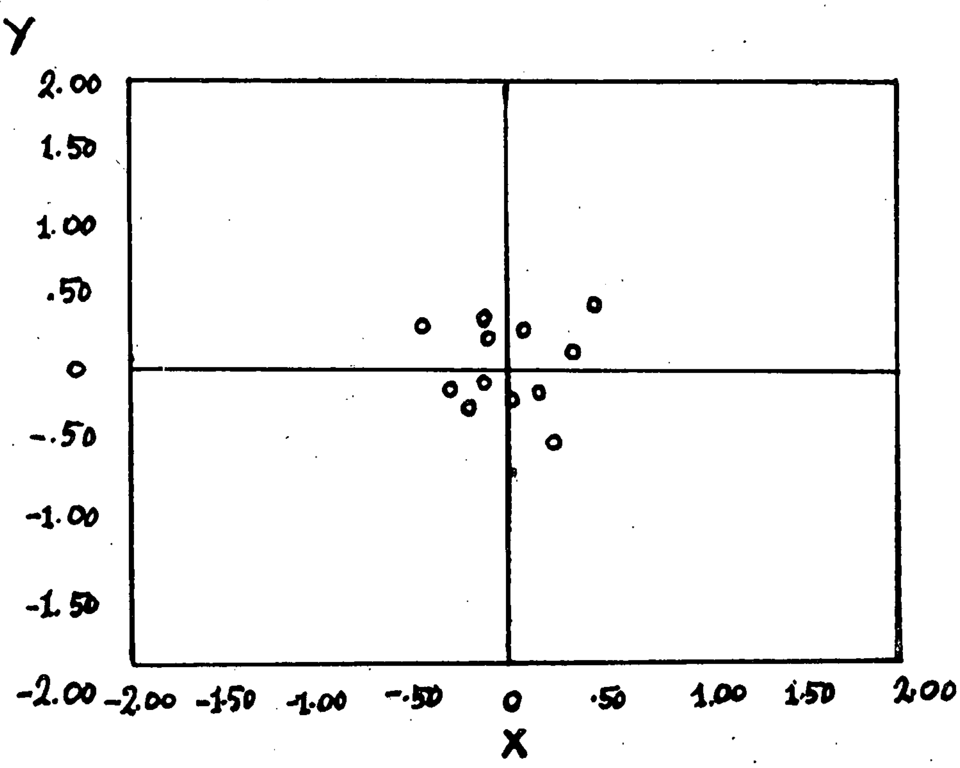
The findings of this Monte Carlo study confirm some of the findings of MacCallum, et al., in 1977 and in 1978. In 1977, MacCallum, et al., found out that number of subjects did not have an effect on the outcome variables. This study did find the same result. In 1978, MacCallum et al., found the superiority of INDSCAL over ALSCAL in terms of the recovery of true structure of the data except for some nonlinear monotonic transformation where ALSCAL outperformed INDSCAL. This study also arrived at the same finding. The difference lies in the nonlinear monotonic transformations that were applied to distort the true distances. As mentioned earlier this study employed five types of transformations: linear, square, cubic, rank, and logarithmic. In terms of the effect of the independent variables (subjects/stimuli/error level/transformation) on the dependent variables, the INDSCAL algorithm proves to be more robust than the ALSCAL algorithm. Specifically, in terms of the recovery of the true distances, INDSCAL recovers true structure of the data better than ALSCAL under large error and at all types of transformation. However, in three nonlinear monotonic distortion of the true distances, i.e., cubic, rank, and logarithmic and at small error ALSCAL did more accurately recover true structure than INDSCAL. Also, in terms of the recovery of the true stimulus configuration, INDSCAL solutions gave a better match of the true configuration than ALSCAL. And finally, in terms of the recovery of subject weights, INDSCAL proves superior to ALSCAL. Figures (10), (11), and (12) show how INDSCAL and ALSCAL differ in the recovery of stimulus coordinates and subject weights under a linearly transformed set of distances. The data that were used in these figures were data representing fifteen subjects and twelve stimuli. Notice that INDSCAL's recovered stimulus space is closer to the true stimulus configuration than ALSCAL's.

In an attempt to explain the different results between the INDSCAL and ALSCAL algorithm, it appears that due to ALSCAL's nonmetric nature, it may be more susceptible to the fitting of random error than INDSCAL, resulting in better fit to the observed data but poorer recovery of true structure (MacCallum, et al., 1978). The primary difference between the fitting algorithm for ordinal ALSCAL and INDSCAL is the optimal scaling phase undertaken by ALSCAL. For the specification of data as subject conditional and at the ordinal level, ALSCAL performs, on each iteration, a unique monotonic transformation of the proximity measures from each subject to improve the fit of the recovered distances to the proximity measures. According to MacCallum, et al., (1978), the optimal scaling phase of ALSCAL may be overly susceptible to the fitting of random error in some cases, i.e., the optimal scaling may be used to improve fit to deviations from linearity, whether these deviations come from systematic distortion or random error. This conditin would be true when the relationship between true distances and observed proximities is approximately linear. In this case, ALSCAL will be overly sensitive to deviations from linearity, while INDSCAL will merely attempt to fit the roughly linear relationship that is distorted by random error. However, if the relatinship between proximities and true distances is severely nonlinear, the relative ability of INDSCAL and ALSCAL to recover the true structure might be reversed, i.e., the optimal scaling phase of ALSCAL would then respond to that strongly nonlinear relationship, thus enhancing recovery of true structure. On the other hand, INDSCAL would linearly fit the model to the severely distorted proximities, hence achieving poorer recovery of true structure.

In the light of these findings, researches who are trying to examine structure in empirical data may be advised that INDSCAL appears to be the preferred method for fitting the weighted Euclidean model. Although most phenomena in life are ordinally scaled, it is most likely the case, as noted by Weeks and Bentler (cited by MacCallum, et al., 1978) that empirical data are roughly linear with respect to the true underlying structure. It is for this reason that the metric INDSCAL excelled the nonmetric ALSCAL.

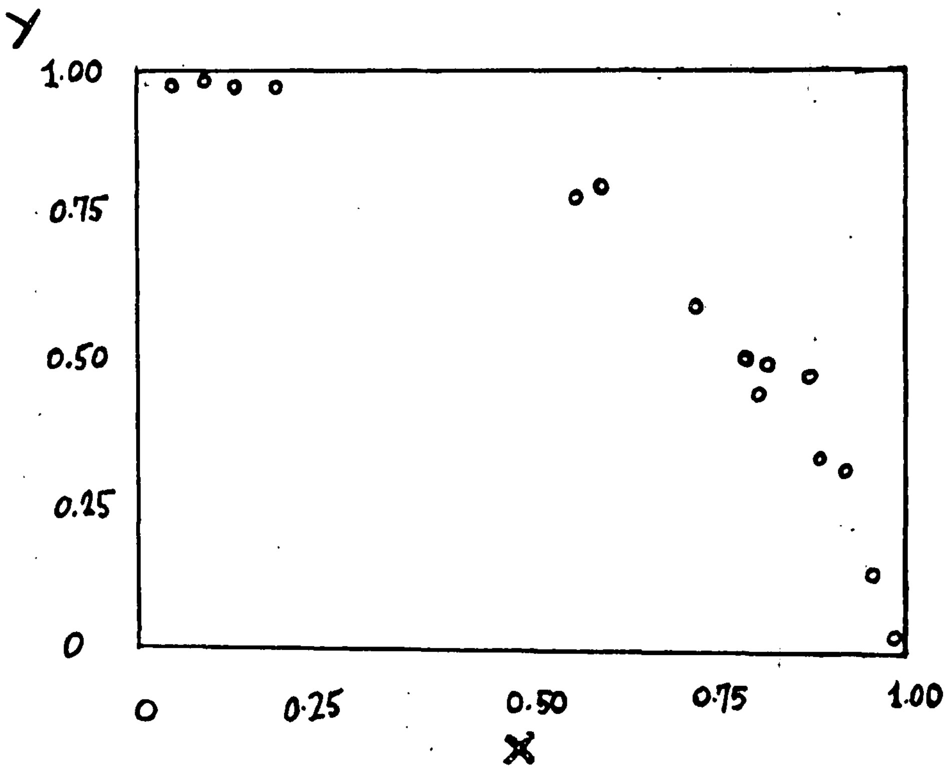


ORIGINAL CONFIGURATION FOR SUBJECT SPACE

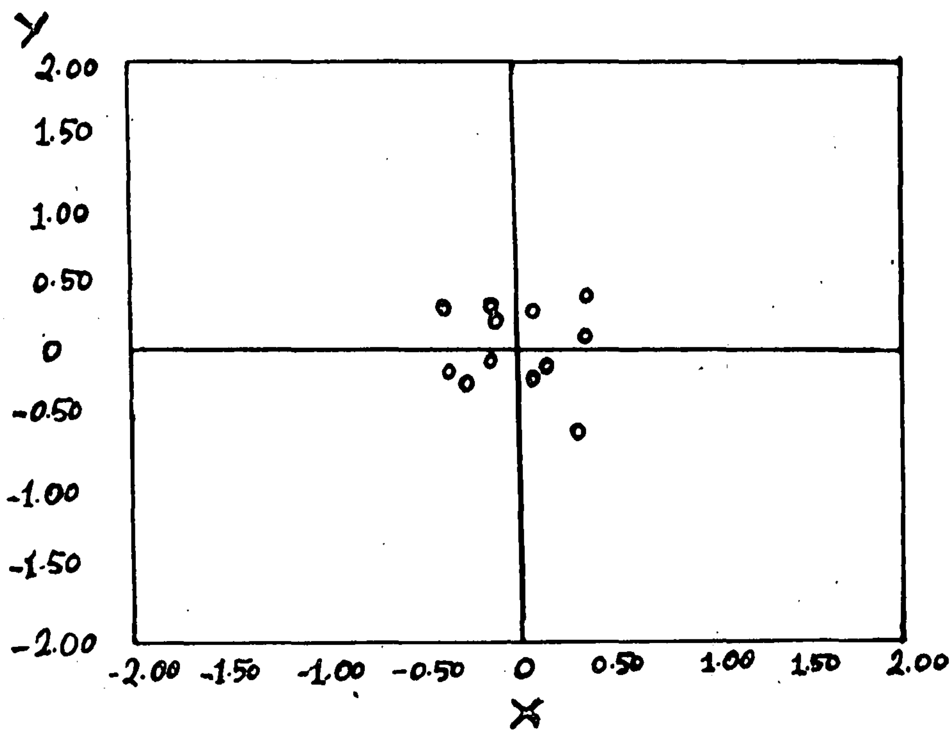


INITIAL STIMULUS SPACE CONFIGURATION

Figure 10: Initial Subject and Stimulus Space

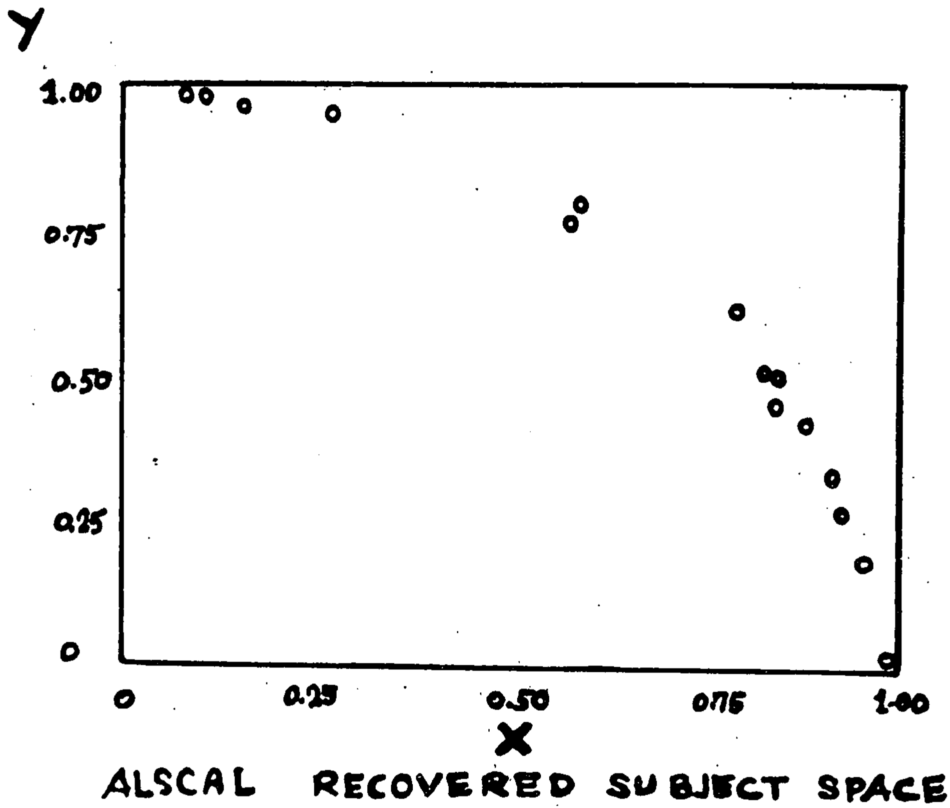


INDSCAL RECOVERED SUBJECT SPACE



INDSCAL RECOVERED STIMULUS SPACE

Figure 11: INDSCAL Recovered Subject and Stimulus Space



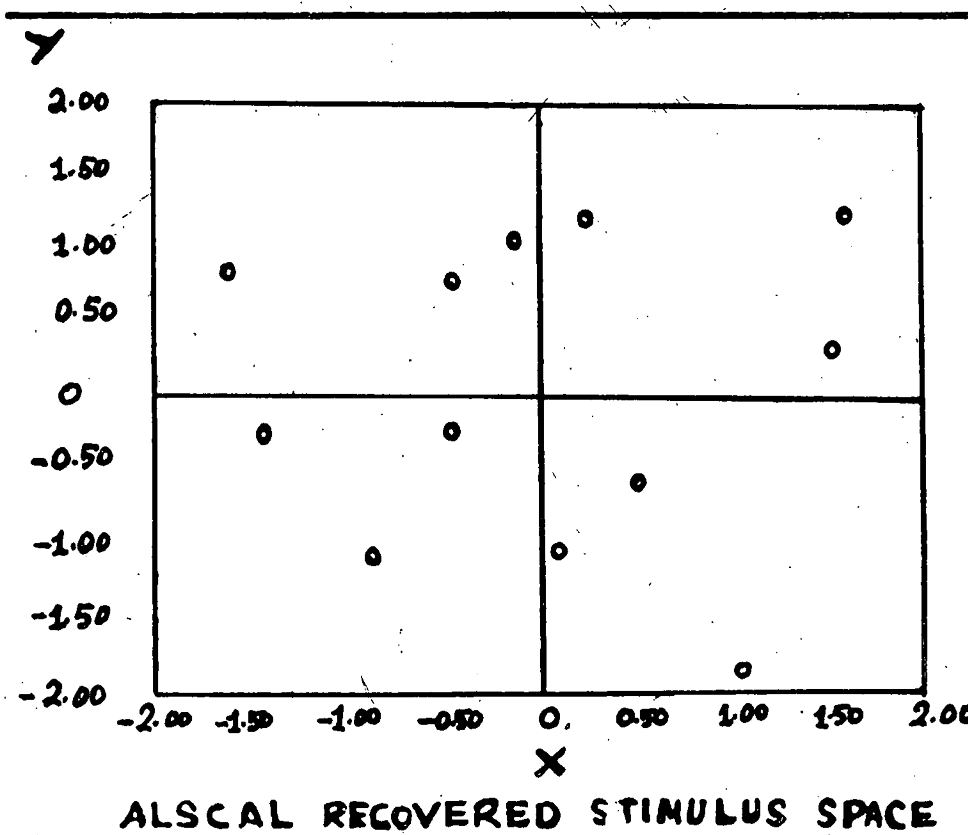


Figure 12: ALSCAL Recovered Subject and Stimulus Space

Table 1

Significance Levels and Omega Squares

From Five-Way Analysis of Variance of RSQ

Source	SS	DF	MS	ω^2	F	Sig of F
BETWEEN DATA SETS						
Within + Residual	.41163	177	.00233			
A (Transf)	4.00500	4	1.00125	.05919	430.53541	<.00001
B (Nsub)	.00049	1	.00049	-	.21107	NS
C (Nstim)	.03486	1	.03486	.00048*	14.98776	.00015
D (Err)	61.90828	1	61.90828	.91698	26620.44165	<.00001
AB	.00156	4	.00039	-	.16779	NS
AC	.17012	4	.04253	.00238*	18.28752	<.00001
AD	.97229	4	.24307	.01426*	104.52045	<.00001
BC	.00040	1	.00040	-	.17130	NS
BD	.00090	1	.00090	-	.38537	NS
CD	.00280	1	.00280	-	1.20205	NS
WITHIN DATA SETS						
Within + Residual	.05549	354	.00016			
E (Method)	.03189	2	.01595	.10841	101.73313	<.00001
AE	.03466	8	.00433	.11626	27.64061	<.00001
BE	.00077	2	.00038	-	2.44368	NS
CE	.00029	2	.00014	-	.91241	NS
DE	.14143	2	.07071	.48258	451.11101	<.00001
ABE	.00101	8	.00013	-	.80209	NS
ACE	.00216	8	.00027	-	1.72160	NS
ADE	.02118	8	.00265	.07021	16.88907	<.00001
BCE	.00034	2	.00017	-	1.09048	NS
BDE	.00006	2	.00003	-	.20338	NS
CDE	.00331	2	.00165	.01077*	10.55540	.00004

* Significant but not meaningful.

Table 1a

Cell Means and Standard Deviations
RSQ

	15 SUBJECTS											
	SMALL ERROR						LARGE ERROR					
	INDSCAL		ALSCAL-ORDINAL		ALSCAL-INTERVAL		INDSCAL		ALSCAL-ORDINAL		ALSCAL-INTERVAL	
	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
12 STIMULI												
(1) Linear	.93464	(.00397)	.93544	(.00519)	.93732	(.00514)	.25660	(.01944)	.20186	(.03310)	.18078	(.03857)
(2) Squared	.92036	(.00509)	.91580	(.00580)	.92198	(.00581)	.42332	(.01238)	.38840	(.01619)	.30390	(.01867)
(3) Cubic	.74812	(.07214)	.78736	(.04520)	.77894	(.05525)	.11980	(.03953)	.09726	(.03305)	.08032	(.03553)
(4) Rank	.76062	(.04926)	.75910	(.03326)	.78878	(.03182)	.11790	(.04246)	.09688	(.03600)	.08410	(.03840)
(5) Log	.76984	(.04860)	.80902	(.03534)	.78814	(.05674)	.21060	(.04659)	.18536	(.05218)	.17854	(.05678)
20 STIMULI												
(1) Linear	.94014	(.00565)	.94250	(.00496)	.94366	(.00447)	.28210	(.01521)	.20852	(.01923)	.19772	(.02270)
(2) Squared	.91176	(.00641)	.90990	(.00385)	.91542	(.00648)	.44512	(.01412)	.41706	(.02340)	.40258	(.02125)
(3) Cubic	.30896	(.04438)	.86234	(.01542)	.84654	(.02277)	.15242	(.02611)	.11292	(.02601)	.10890	(.02848)
(4) Rank	.82442	(.03229)	.86382	(.01397)	.86058	(.01753)	.15324	(.02766)	.11284	(.02804)	.10746	(.03057)
(5) Log	.72426	(.04904)	.30650	(.01347)	.75039	(.03912)	.15882	(.02337)	.13494	(.03074)	.13074	(.03332)
	30 SUBJECTS											
12 STIMULI												
(1) Linear	.93688	(.00137)	.93680	(.00276)	.93922	(.00128)	.26634	(.01776)	.20352	(.02289)	.18062	(.03216)
(2) Squared	.91070	(.00544)	.90848	(.00447)	.91390	(.00398)	.42584	(.01625)	.40442	(.02161)	.37514	(.01327)
(3) Cubic	.74420	(.06467)	.79920	(.02552)	.79030	(.03284)	.12470	(.03200)	.09660	(.03271)	.08448	(.03375)
(4) Rank	.77202	(.02388)	.79052	(.01362)	.79376	(.01249)	.12356	(.02869)	.09254	(.03247)	.07706	(.03553)
(5) Log	.76270	(.04371)	.82086	(.02558)	.78862	(.04971)	.21878	(.04143)	.18542	(.04753)	.16626	(.05720)
20 STIMULI												
(1) Linear	.34114	(.00126)	.94412	(.00127)	.94456	(.00057)	.28394	(.01377)	.21908	(.01038)	.20162	(.01127)
(2) Squared	.91426	(.00682)	.91156	(.00512)	.91740	(.00581)	.45222	(.00508)	.43076	(.00819)	.39714	(.00635)
(3) Cubic	.81880	(.03185)	.86182	(.01029)	.84176	(.01698)	.15792	(.02408)	.11674	(.02001)	.11228	(.02054)
(4) Rank	.82016	(.03466)	.86260	(.01083)	.85656	(.01614)	.15626	(.02228)	.11682	(.01810)	.11138	(.01871)
(5) Log	.71852	(.05950)	.77828	(.03920)	.69780	(.06645)	.17146	(.01725)	.14470	(.01101)	.14020	(.01212)

Table 2

<u>Significance Levels and Omega Squares</u>
<u>From Five-Way Analysis of Variance of d</u>

Source	SS	DF	MS	ω^2	F	Sig of F
<u>BETWEEN DATA SETS</u>						
Within + Residual	2.93609	177	.01659			
A (Transf)	.13866	4	.03466	-	2.08975	NS
B (Nsub)	.01249	1	.01249	-	.75283	NS
C (Nstim)	.35120	1	.35120	.09294	21.17216	<.00001
D (Err)	.05474	1	.05474	-	3.30012	NS
AB	.00100	4	.00025	-	.01503	NS
AC	.03992	4	.00998	-	.60163	NS
AD	.00802	4	.00201	-	.12087	NS
BC	.03657	1	.03657	-	2.20448	NS
BD	.00161	1	.00161	-	.09719	NS
CD	.00339	1	.00339	-	.20440	NS
<u>WITHIN DATA SETS</u>						
Within + Residual	1.59332	354	.00450			
E (Method)	5.65547	2	2.82773	.76529	628.25886	<.00001
AE	.05353	8	.00669	-	1.48670	NS
BE	.00708	2	.00354	-	.78617	NS
CE	.02740	2	.01370	.00310*	3.04423	.04988
DE	.01272	2	.00636	-	1.41322	NS
ABE	.00645	8	.00081	-	.17902	NS
ACE	.01146	8	.00143	-	.31631	NS
ADE	.00512	8	.00064	-	.14233	NS
BCE	.00206	2	.00103	-	.22912	NS
BDE	.00198	2	.00099	-	.21994	NS
CDE	.00299	2	.00149	-	.33184	NS

* Significant but not meaningful.

<u>Note.</u> Higher order interactions not appearing in the table were suppressed because they explained less than .2% of the variance.

Table 2a

Cell Means and Standard Deviations

δ

	15 SUBJECTS											
	SMALL ERROR						LARGE ERROR					
	INDSCAL		ALSCAL-ORDINAL		ALSCAL-INTERVAL		INDSCAL		ALSCAL-ORDINAL		ALSCAL-INTERVAL	
	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
12 STIMULI												
(1) Linear	.01226	(.00261)	.24348	(.13949)	.24184	(.14008)	.05890	(.00917)	.25208	(.10192)	.24478	(.09651)
(2) Squared	.02072	(.00264)	.25956	(.15249)	.26296	(.14941)	.03518	(.00688)	.26884	(.12854)	.26738	(.12599)
(3) Cubic	.06160	(.01596)	.28536	(.13986)	.28412	(.13906)	.14916	(.06793)	.31134	(.10480)	.28970	(.10650)
(4) Rank	.05044	(.01652)	.27644	(.14697)	.30284	(.13952)	.12804	(.06689)	.30280	(.10345)	.30946	(.10474)
(5) Log	.05542	(.01143)	.27080	(.14198)	.26600	(.13778)	.09300	(.01545)	.28694	(.11101)	.28386	(.11917)
20 STIMULI												
(1) Linear	.00952	(.00104)	.22668	(.12416)	.22644	(.12396)	.03982	(.00370)	.24304	(.09070)	.24536	(.08993)
(2) Squared	.01828	(.00122)	.22656	(.12520)	.22814	(.12017)	.02448	(.00370)	.23390	(.11437)	.23494	(.11386)
(3) Cubic	.04412	(.00849)	.22908	(.11669)	.22970	(.11570)	.07730	(.01429)	.23090	(.08785)	.23752	(.09144)
(4) Rank	.04108	(.00956)	.23642	(.11614)	.23704	(.11461)	.07854	(.01207)	.26908	(.03694)	.26336	(.04893)
(5) Log	.05558	(.00994)	.22166	(.11351)	.22382	(.10130)	.08712	(.01554)	.23910	(.07821)	.23398	(.07908)
	30 SUBJECTS											
12 STIMULI												
(1) Linear	.01018	(.00160)	.28420	(.15543)	.28340	(.15677)	.04460	(.00820)	.30416	(.11363)	.29926	(.10856)
(2) Squared	.01928	(.00175)	.29042	(.16127)	.29138	(.15689)	.02470	(.00439)	.30000	(.13906)	.29830	(.14565)
(3) Cubic	.11294	(.11571)	.29534	(.14071)	.29728	(.14080)	.15228	(.10052)	.31414	(.07584)	.31828	(.07692)
(4) Rank	.09994	(.12103)	.32954	(.05133)	.33096	(.05233)	.15822	(.12716)	.34394	(.04157)	.34114	(.03604)
(5) Log	.05672	(.00938)	.29364	(.15422)	.29944	(.14834)	.08382	(.01539)	.32032	(.11241)	.32010	(.11757)
20 STIMULI												
(1) Linear	.00810	(.00100)	.22248	(.12404)	.22280	(.12417)	.02752	(.00566)	.23726	(.09783)	.23764	(.09649)
(2) Squared	.01690	(.00160)	.22702	(.12609)	.22964	(.12188)	.01688	(.00277)	.23020	(.11548)	.22886	(.11302)
(3) Cubic	.03824	(.00606)	.22982	(.11780)	.23112	(.11662)	.05556	(.01193)	.22720	(.09095)	.22870	(.09335)
(4) Rank	.03728	(.00698)	.22854	(.11808)	.23004	(.11546)	.05658	(.01119)	.25038	(.06709)	.24274	(.06780)
(5) Log	.04746	(.00845)	.22598	(.11637)	.23044	(.10147)	.06158	(.01082)	.23770	(.08061)	.23322	(.09016)

Table 3

Marginal Means (RSQ, δ, φ)

	RSQ			δ					
	COUNT	$\bar{X}$	SD	COUNT	$\bar{X}$	SD	COUNT	$\bar{X}$	SD
Transf									
Linear	120	.5816	(.3608)	120	.1761	(.1399)	120	.0874	(.0719)
Squared	120	.6624	(.2539)	120	.1773	(.1488)	120	.0934	(.0816)
Cubic	120	.4605	(.3512)	120	.2055	(.1277)	120	.0843	(.0582)
Rank	120	.4622	(.3537)	120	.2144	(.1283)	120	.0885	(.0564)
Log	120	.4684	(.3050)	120	.1970	(.1283)	120	.0855	(.0534)
Nsub									
Fifteen	300	.5261	(.3378)	300	.1895	(.1306)	300	.0865	(.0625)
Thirty	300	.5279	(.3356)	300	.1986	(.1399)	300	.0892	(.0672)
Nstim									
Twelve	300	.5194	(.3347)	300	.2182	(.1456)	300	.0820	(.0601)
Twenty	300	.5346	(.3386)	300	.2698	(.1196)	300	.0936	(.0689)
Err									
Small	300	.8482	(.07797)	300	.1845	(.1439)	300	.0919	(.0793)
Large	300	.2058	(.1167)	300	.2036	(.1256)	300	.0838	(.0459)
Method									
INDSCAL	200	.5346	(.3174)	200	.05673	(.0540)	200	.0326	(.0486)
ALSCAL-ORDINAL	200	.5293	(.3449)	200	.26267	(.1095)	200	.1175	(.0558)
ALSCAL-INTERVAL	200	.5172	(.3476)	200	.26270	(.1084)	200	.1133	(.0509)

Table 4

Significance Levels and Omega Squares
From Five-Way Analysis of Variance of ϕ

Source	SS	DF	MS	ω^2	F	Sig of F
BETWEEN DATA SETS						
Within + Residual	.76312	177	.00431			
A (Transf)	.00591	4	.00148	-	.34252	NS
B (Nsub)	.00112	1	.00112	-	.25974	NS
C (Nstim)	.02009	1	.02009	.01728*	4.65922	.03223
D (Err)	.00982	1	.00982	-	2.27721	NS
AB	.06054	4	.01513	.04742	3.51024	.00873
AC	.01675	4	.00419	-	.97117	NS
AD	.01720	4	.00430	-	.99763	NS
BC	.00926	1	.00926	-	2.14848	NS
BD	.00324	1	.00324	-	.75238	NS
CD	.00147	1	.00147	-	.34121	NS
WITHIN DATA SETS						
Within + Residual	.46258	354	.00131			
E (Method)	.91597	2	.45799	.56663	350.48285	<.00001
AE	.03221	8	.00403	.01672*	3.08130	.00227
BE	.02843	2	.01422	.01681*	10.88005	.00003
CE	.00258	2	.00129	-	.98781	NS
DE	.07969	2	.03984	.04856	30.49185	<.00001
ABE	.02298	8	.00287	.01100*	2.19864	.02703
ACE	.04084	8	.00510	.02206*	3.90668	.00019
ADE	.00787	8	.00098	-	.75288	NS
BCE	.01771	2	.00885	.01016*	6.77638	.00129
BDE	.00091	2	.00045	-	.34643	NS
CDE	.00114	2	.00057	-	.43755	NS

* Significant but not meaningful.

Note. Higher order interactions not appearing in the table were suppressed because they explained less than .2% of the variance.

Table 4a

Cell Means and Standard Deviations

	Small Error						Large Error					
	INDSCAL		ALSCAL-ORDINAL		ALSCAL-INTERVAL		INDSCAL		ALSCAL-ORDINAL		ALSCAL-INTERVAL	
	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
15 SUBJECTS												
12 STIMULI												
(1) Linear	.00126	(.00083)	.13056	(.07582)	.12728	(.07337)	.02772	(.01590)	.09688	(.02731)	.09670	(.02132)
(2) Squared	.00054	(.00017)	.13166	(.08426)	.13344	(.08352)	.01016	(.00434)	.10990	(.05018)	.11100	(.04311)
(3) Cubic	.01506	(.01251)	.07190	(.04170)	.06562	(.03499)	.08698	(.07311)	.06972	(.00525)	.06766	(.00558)
(4) Rank	.00994	(.00840)	.09266	(.05812)	.10382	(.05720)	.04748	(.04334)	.10428	(.00751)	.10644	(.00507)
(5) Log	.01530	(.01174)	.12140	(.07600)	.10108	(.04919)	.04986	(.03351)	.09108	(.04671)	.10476	(.01562)
20 STIMULI												
(1) Linear	.07620	(.10526)	.11174	(.06686)	.11048	(.06617)	.07992	(.09052)	.07494	(.01738)	.07656	(.01401)
(2) Squared	.07700	(.10641)	.13434	(.07572)	.14714	(.08881)	.07718	(.10178)	.12102	(.06286)	.11468	(.05602)
(3) Cubic	.00628	(.00323)	.11832	(.06674)	.10598	(.05697)	.03384	(.01225)	.08996	(.01116)	.08976	(.00889)
(4) Rank	.00464	(.00292)	.12440	(.06867)	.12190	(.06543)	.03292	(.01141)	.12080	(.01374)	.12164	(.01142)
(5) Log	.07594	(.08385)	.17408	(.04660)	.13524	[illegible]	.09028	(.06711)	.11818	(.00259)	.11970	(.00439)
30 SUBJECTS												
12 STIMULI												
(1) Linear	.00104	(.00011)	.15172	(.08481)	.15014	(.08400)	.02524	(.00444)	.09698	(.02715)	.09648	(.01967)
(2) Squared	.00058	(.00036)	.13850	(.08482)	.13794	(.08454)	.01012	(.00157)	.11184	(.05300)	.09516	(.04124)
(3) Cubic	.04252	(.06425)	.13294	(.06949)	.12272	(.05935)	.07482	(.03641)	.10840	(.00746)	.11136	(.00494)
(4) Rank	.04182	(.07566)	.14988	(.01169)	.14658	(.01204)	.07726	(.04191)	.10232	(.00232)	.10192	(.00243)
(5) Log	.01274	(.00543)	.10846	(.06275)	.07628	(.03793)	.04372	(.01045)	.07538	(.00588)	.07500	(.00620)
20 STIMULI												
(1) Linear	.00084	(.00030)	.16284	(.09295)	.15974	(.09123)	.01702	(.00476)	.11326	(.02987)	.11304	(.02671)
(2) Squared	.00030	(.00020)	.16380	(.09567)	.17462	(.09524)	.00510	(.00159)	.12676	(.06772)	.11814	(.06010)
(3) Cubic	.00686	(.00324)	.15172	(.08561)	.13968	(.07533)	.03334	(.00801)	.13874	(.01768)	.13820	(.01583)
(4) Rank	.00490	(.00291)	.12854	(.06959)	.12656	(.06713)	.03100	(.00646)	.11082	(.00541)	.11086	(.00572)
(5) Log	.01618	(.00634)	.12980	(.06971)	.09634	(.03098)	.04082	(.00882)	.09126	(.00617)	.09010	(.00758)

Recommendations

This researcher feels that there are two areas to explore for further research along Multidimensional Scaling analysis. First, since the amount of error has been shown repeatedly to be the most important factor in the "goodness" of multidimensional scaling solutions, attention should be focused on empricial studies of the variables affecting measurement error; e.g., human error as method of collecting data, subject familiarity with stimuli, subject motivation, difficulty of judgments/or subject's perceptions of MDS is that real data genrally has some degree of error in it. Thus, the question arises: "How much error can we tolerate and yet expect to reconstruct the geometric representation to an acceptable degree?"

Second, since this study tried to look at factors that distinguished metric from nonmetric MDS, still there should be other factors or variables that have not yet been explored. Perhaps some kind of transformation of the true distances (i.e., a higher-order polynomial of degree 4, 5, or 6; or a product of this higher-order polynomial and a trigonometric function) will do.

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